TEMPERATURE DISTRIBUTION IN A COMPOSITE ROD WITH A STEP CHANGE IN THE CONDITIONS OF HEAT TRANSFER

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UDC 536.24.01

A method is proposed for calculating the steady-state temperature field of a model pipe fitting. The results are compared with test data.

We consider a situation often encountered in pipe fitting design, where the lower part of the rod structure is thermally insulated while the upper part is cooled by natural convection and radiation (Fig. 1). At low values of the Biot number (Bi $\simeq$ 0.02-0.05) the problem can be reduced to solving a system of one-dimensional heat transmission equations for the individual geometrical components which are characterized by different conditions of heat transfer. For thermally insulated components these equations are

$$\frac{d}{d\xi}\left[\lambda(\vartheta_i)\frac{d\vartheta_i}{d\xi}\right] = m_i q(\vartheta_i) \quad (i = 1,\ 2,\ 3), \tag{1}$$

for components cooled by laminar natural convection these equations are

$$\frac{d}{d\xi}\left[\lambda(\vartheta_i)\frac{d\vartheta_i}{d\xi}\right] = C_i\left[\frac{\mathrm{Nu}_x}{(\mathrm{Gr}_x/4)^{\frac{1}{4}}}\right]_i \vartheta_i^{\frac{5}{4}}\xi^{-\frac{1}{4}} + S_i\{[1 + (\vartheta_0 - 1)\vartheta_i]^5 - 1\}$$
$$(i = 4,\ 5), \tag{2}$$
$$b_{i-1} \leqslant \xi \leqslant b_i \quad (i = 1,\ 2,\ \ldots,\ 5),$$

with the radiation energy taken as proportional to the fifth power of the absolute temperature [1], and with the magnitude of the generalized heat transfer coefficient $\mathrm{Nu}_x/(\mathrm{Gr}_x/4)^{1/4}$ varying over the height of the heat transfer surface.

The solutions to Eqs. (1) and (2) for individual components join, with respect to temperature and thermal flux, according to the respective boundary conditions:

$$\text{at } \xi = b_0 = 0 \quad \vartheta_1 = 1; \quad \text{at } \xi = b_1 \quad \vartheta_1 = \vartheta_2,$$
$$-\lambda(\vartheta_1)\frac{d\vartheta_1}{d\xi} = -k_1\lambda(\vartheta_2)\frac{d\vartheta_2}{d\xi} + k_2\vartheta_1;$$
$$\text{at } \xi = b_2 \quad \vartheta_2 = \vartheta_3,\ \lambda(\vartheta_2)\frac{d\vartheta_2}{d\xi} = k_3\lambda(\vartheta_3)\frac{d\vartheta_3}{d\xi} - k_4[\vartheta_2(b_2) - \vartheta_4(b_4)]^{\frac{1}{4}}\frac{d\vartheta_3}{d\xi};$$
$$\text{at } \xi = b_3 \quad \vartheta_3 = \vartheta_4,\ \frac{d\vartheta_3}{d\xi} = \frac{d\vartheta_4}{d\xi}; \quad \text{at } \xi = b_4 \quad \vartheta_4 = \vartheta_5,\ \lambda(\vartheta_4)\frac{d\vartheta_4}{d\xi} \tag{3}$$
$$+ k_5[\vartheta_2(b_2) - \vartheta_4(b_4)]^{\frac{1}{4}}\frac{d\vartheta_4}{d\xi} = k_6\lambda(\vartheta_5)\frac{d\vartheta_5}{d\xi} - k_7\left[\frac{\mathrm{Nu}_x}{(\mathrm{Gr}_x/4)^{\frac{1}{4}}}\right]_5 \xi^{-\frac{1}{4}}\vartheta_5^{\frac{5}{4}};$$
$$\text{at } \xi = b_5 = 1 \quad -\lambda(\vartheta_5)\frac{d\vartheta_5}{d\xi} = k_8\left[\frac{\mathrm{Nu}_x}{(\mathrm{Gr}_x/4)^{\frac{1}{4}}}\right]_5 \xi^{-\frac{1}{4}}\vartheta_5^{\frac{5}{4}} + k_9\{[1 + (\vartheta_0 - 1)\vartheta_5]^5 - 1\}.$$

M. I. Kalinin Polytechnic Institute, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 23, No. 5, pp. 884-889, November, 1972. Original article submitted February 21, 1972.

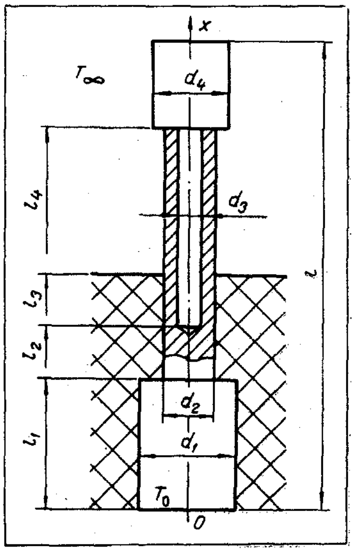

Fig. 1

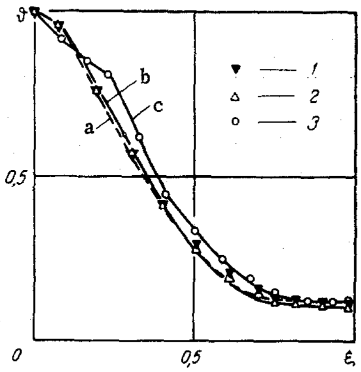

Fig. 2

Fig. 1. Rod with thermal insulation, natural convection, and radiation.

Fig. 2. Comparison between test data and calculations ($Pr_\infty = 0.7$, $Nu_x/(Gr_x/4)^{1/4} = 0.5$, $c_k = 0.1$, $d_1 = 0.065$ m, $d_2 = 0.035$ m, $d_3 = 0.014$ m, $d_4 = 0.062$ m). (a) $T_0 = 470$°K, $T_\infty = 298$°K, $l = 0.387$ m, $b_1 = 0.09$, $b_2 = 0.155$, $b_3 = 0.4$, $b_4 = 0.71$ (1). (b) $T_0 = 673$°K, $T_\infty = 298$°K, and the same dimensions (2). (c) $T_0 = 428$°K, $T_\infty = 296$°K, $l = 0.464$ m, $b_1 = 0.241$, $b_2 = 0.295$, $b_3 = 0.414$, $b_4 = 0.759$ (3).

The thermal conductivity of the material is assumed, in practical calculations, to be a linear function of the temperature:

$$\lambda(\vartheta_i) = 1 + cT_\infty \left[1 + (\vartheta_0 - 1)\vartheta_i\right]. \tag{4}$$

The effect of convection inside enclosed cavities is accounted for by an equivalent thermal conductivity [2]:

$$\lambda_э = \varepsilon_k \left[\vartheta_2(b_2) - \vartheta_4(b_4)\right]^{\frac{1}{4}}. \tag{5}$$

The heat losses through the insulation are proportional to the temperature:

$$q(\vartheta_i) = \vartheta_i / R_i. \tag{6}$$

The calculation of natural convection is made difficult by the fact that, with an exponential temperature distribution along the rod height, the temperature profile of the boundary layer is not self-adjoint and the value of the $Nu_x/(Gr_x/4)^{1/4}$ complex decreases along the length coordinate [3]. In order to evaluate the thermal flux from the surface, one may use the mean value of $Nu_x/(Gr_x/4)^{1/4}$ obtained earlier [3] for various values of the Prandtl number, but the accuracy of the calculated temperature distribution improves appreciably if the linear approximation

$$\left[Nu_x/(Gr_x/4)^{\frac{1}{4}}\right]_i = \left[Nu_x/(Gr_x/4)^{\frac{1}{4}}\right]_0 (1 - c_k\zeta) \tag{7}$$

is used, where the value of $[Nu_x/(Gr_x/4)^{1/4}]_0$ is taken from the self-adjoint solutions to the equations of natural convection [4] and where coefficient c_k is based on the decrease in the thermal flux [3]. If the initial point of the boundary layer at the thermally uninsulated rod segment does not lie at the origin of coordinates (Fig. 1, i = 4, 5), then $\zeta = \xi - b_3$ and the linear dimension in the local Nusselt and Grashof numbers is $x - l_3$.

For a numerical solution of the problem, each of the five equations (1), (2) is reduced to a system of two first-order equations with respect to the thermal flux $u_i = -\lambda(\vartheta_i)d\vartheta_i/d\xi$ and temperature $\vartheta_i(\xi)$:

$$d\vartheta_i/d\xi = -u_i/\lambda(\vartheta_i) \quad (i = 1, 2, \ldots, 5),$$

$$\frac{du_i}{d\xi} = \begin{cases} -m_i\vartheta_i/R_i & (i=1,\ 2,\ 3), \\ -C_i\left[\dfrac{\mathrm{Nu}_x}{\left(\dfrac{\mathrm{Gr}_x}{4}\right)^{\frac{1}{4}}}\right]_i \vartheta_i^{\frac{5}{4}}\xi^{-\frac{1}{4}} - S_i\left\{\left[1+(\vartheta_0-1)\vartheta_i\right]^5-1\right\} & (i=4,\ 5) \end{cases}$$

$$b_{i-1} \leqslant \xi \leqslant b_i \quad (i=1,\ 2,\ \ldots,\ 5). \tag{8}$$

Here the boundary conditions retain their form (3), with $d\vartheta_i/d\xi$ replaced by $-u_i/\lambda(\vartheta_i)$.

The boundary-value problem (8), (3) can be solved as a Cauchy problem, if the values of functions $u_1(0)$ and $\vartheta_5(1)$ are given at both respective end points of the $0 \le \xi \le 1$ interval. Since these values are not known at the start of calculations, they must be assumed arbitrarily to the first approximation. The sequence of integrations over the segments of the $0 \le \xi \le 1$ interval is governed by the singularity of system (8) at $\xi = b_3$ and by the singularities in the boundary conditions: at $\xi = b_2$ and $\xi = b_4$ the quantity $[\vartheta_2(b_2) - \vartheta_4(b_4)]^{1/4}$ can be calculated only after the equations have been solved for the second segment ($b_1 \le \xi \le b_2$) and the fourth segment ($b_3 \le \xi \le b_4$). For this reason, we have adopted the following scheme: Eqs. (8) are integrated from $\xi = 0$ over the first segment ($0 \le \xi \le b_1$) and the second segment ($b_1 \le \xi \le b_2$), then in the reverse direction from $\xi = 1$ over the fifth segment ($b_4 \le \xi \le b_5$) and the fourth segment ($b_3 \le \xi \le b_4$), which makes it possible to calculate $[\vartheta_2(b_2) - \vartheta_4(b_4)]^{1/4}$ and, after returning to point $\xi = b_2$, to integrate over the third segment ($b_2 \le \xi \le b_3$). Reverse integration over the fifth and the fourth segment ($i = 4, 5$) makes it possible to remove the singularity of Eq. (8) at $\xi = b_3$ and to approach the singular point as closely as desired. During transition through the end points b_i, the values of u_i and u_{i+1} necessary for further integration are determined from condition (3). Inasmuch as $u_1(0)$ and $\vartheta_5(1)$ have been selected arbitrarily, functions $u_3(\xi)$ and $u_4(\xi)$ as well as functions $\vartheta_3(\xi)$ and $\vartheta_4(\xi)$ do not meet at first when integration is performed in opposite directions from the two endpoints of the $0 \le \xi \le 1$ interval and, therefore, they must be joined now at point $\xi = b_3$. The real values of $u_1(0)$ and $\vartheta_5(1)$ at which $\vartheta_3(\xi) = \vartheta_4(\xi)$ and $u_3(\xi) = u_4(\xi)$ at point b_3 are found by the Newton method. System (8) is integrated over each segment by the Runge–Kutta scheme. The advantage of this entire procedure is its fast convergence (4-5 approximations) and little machine time required (approximately 1 min for solving the problem on a BÉSM-4 computer).

The results of such a solution were checked experimentally on vertical models made of grade 0Kh18-N10T steel. The lower part of the models was insulated with asbestos, while the protruding upper part was cooled by natural convection in air. The base was heated in a laboratory shaft furnace, but convective currents from the heater were kept isolated by a protective shield with a special cooling system which maintained room temperature at the shield surface. We examined the steady-state temperature distributions along the height and across a section of the models, also inside the enclosed cavities. The temperature at 15 control sections was measured with Chromel–Alumel thermocouples welded at the periphery and at the center of a section by the capacitor welding technique. The temperature nonuniformity over a cross section in the base part did not exceed 1°C. The temperature field of the enclosed cavity was searched with a microthermocouple inserted through the wall along capillary porcelain insulators tightly mounted in drilled holes. The thermocouple probe was moved across a cavity section by means of a micrometer coordinator screw with a 0.01 mm precision. The measured temperature distribution along the cavity height almost coincided with that along the model wall, while the temperature across a cavity section above the thermal insulation increased toward the center by 2–4°C.

The calculated results are compared with test values in Fig. 2. The maximum discrepancy between them does not exceed 3°C within the upper zone of the rod cavity: heat convection inside the cavity is accounted for only in the boundary conditions (3), where it affects the second digit in the value of the derivative $d\vartheta/d\xi$ – important for orienting the temperature curve to join together the solutions.

The proposed method reduces the computation time for the design of a pipe fitting [5] to several minutes, which is of practical interest in an analysis of structural variants during the design stage.

NOTATION

T is the absolute temperature;
P_i is the section perimeter;
F_i cross-sectional area;
l_i is the section boundary;

R_i is the thermal resistance to heat transfer;

$\vartheta = (T - T_\infty)/(T_0 - T_\infty)$;

$\vartheta_0 = T_0/T_\infty$ are the dimensionless temperatures;

$\xi = x/l$ is the dimensionless coordinate;

ζ is the dimensionless ordinate in the boundary layer;

c_k is the coefficient accounting for change of thermal flux at the surface;

$Nu_x = \alpha_x(x - l_3)/\lambda$ is the Nusselt number;

$Gr_x = g\beta(T - T_\infty)(x - l_3)^3/\nu^2_\infty$;

$Gr_0 = g\beta(T_0 - T_\infty)l^3/\nu^2_\infty$;

$Gr_V = g\beta_V(T_0 - T_\infty)(l_4 - l_2)^3/\nu^2_V$ are the Grashof numbers;

$Bi = \alpha d/\lambda_c$ is the Biot number;

$Pr = \nu/a$ is the Prandtl number;

$m_i = P_i l^2/F_i\lambda_c$;

$C_i = (P_i l\lambda_\infty/F_i\lambda_c)(Gr_0/4)^{1/4}$;

$S_i = [P_i l^2 a_s/F_i\lambda_c(T_0 - T_\infty)](T_\infty/100)^5$;

$k_1 = F_2/F_1$;

$k_2 = (F_1 - F_2)l/F_1\lambda_c R_1$;

$k_3 = (F_2 - F_3)/F_2$;

$k_4 = (F_3/F_2)\varepsilon_k$;

$k_5 = [F_3/(F_2 - F_3)]\varepsilon_k$;

$k_6 = F_4/(F_2 - F_3)$;

$k_7 = [(F_4 - F_3)\lambda_\infty/(F_2 - F_3)\lambda_c]0.7(Gr_0/4)^{1/4}$;

$k_8 = (\lambda_\infty/\lambda_c)1.3(Gr_0/4)^{1/4}$;

$k_9 = [a_s l/\lambda_c(T_0 - T_\infty)](T_\infty/100)^5$;

$\varepsilon_k = 0.18(\lambda_V/\lambda_c)(Gr_V \cdot Pr_V)^{1/4}$ are constant coefficients in Eqs. (1), (2), and (3). For grade 0Kh18N10T steel at $290°K \leq T \leq 900°K$ $\lambda_r = 8.99$ W/m·°K, $c = 0.00207$ (°K)$^{-1}$, $a_s = 0.2465$ W/m^2·°K^5.

Subscripts

x denotes the local value, function of the coordinate;
0 denotes the original of coordinates;
∞ denotes the ambient medium;
h denotes the inside cavity;
r denotes the rod.

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